ENERGY EFFICIENT REDUNDANCY

JEFFREY A. BARNETT

1. Summary

This note presents an alternative and simplified[1] formulation of a problem introduced by Dan Mosse and Rami Melham. The objective is to implement a triply redundant computation while minimizing energy expenditure. The strategy is to execute two of the three identical processes relatively rapidly. If the two processes agree on their results, all computation ceases. If they do not agree, the third process, executing at a slower rate, is sped up to complete its results by the deadline. The optimization result is shown to have the following form:

$$t_1 = f(p) \cdot t_2 \qquad\qquad c_1 = g(p) \cdot c,$$

where p is the probability that processes 1 and 2 disagree, t_2 is the hard realtime deadline, $t_1 \leq t_2$ is the rendezvous time for processes 1 and 2, c is the number of cycles executed by each of the three processes, and $c_1 \leq c$ is the number of cycles process 3 executes by t_1. Interestingly, $f(1/3) = g(1/3) = 1$ which means that full symmetric redundancy is as efficient as any seemingly more clever scheme when $p \geq 1/3$.

Section 2 presents the math models, Section 3 does the optimization, and Section 4 analyzes the results.

2. Resource and Redundancy Models

Energy/Time Model:

- $t = c/v$, where t is the time to compute c cycles at voltage setting v.
- $e = cv$, where e is the energy expenditure to compute c cycles at voltage setting v.
- No energy is expended when no computation is being done.

Redundancy and Application Models:

- Three identical processes that each execute c cycles.
- Computation starts at time 0.

[1]Read as less realistic.

- Error-free results, if available, must be provided by time t_2, the hard deadline.
- Processes 1 and 2 will finish all c cycles by time $t_1 \leq t_2$.
- At time t_1, process 3 will have completed execution of $0 \leq c_1 \leq c$ cycles.
- If processes 1 and 2 produce the same result at time t_1, that result is used and all processing halts.
- If process 1 and 2 disagree, they halt but process 3 executes its remaining $c - c_1$ cycles in the period between t_1 and t_2. Its results are combined/voted with the other results to determine the system's solution.
- Processes 1 and 2 fail to agree with probability p.

Figure 1 shows the time of execution of the three processes and the cycles executed in each period.

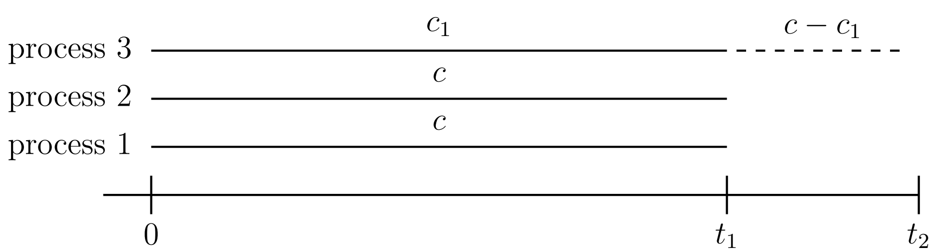

FIGURE 1. Time lines and cycles executed by periods.

3. Optimization

Since $t = c/v$, $v = c/t$ is the voltage setting necessary to execute c cycles in time t. Since $e = cv$, $e = c^2/t$ is the energy expenditure necessary to execute c cycles in time t. Hence,

$$e_1 = \frac{c^2}{t_1} \qquad e_2 = \frac{c^2}{t_1} \qquad e_{31} = \frac{c_1^2}{t_1} \qquad e_{32} = \frac{(c - c_1)^2}{t_2 - t_1},$$

where e_1 is the energy expenditure of process 1, e_2 is the energy expenditure of process 2, e_{31} is the energy expenditure of process 3 up to time t_1, and e_{32} is the energy expenditure of process 3 between times t_1 and t_2. Since the latter expense is only incurred, with probability p, if processes 1 and 2 disagree, the *expected* total energy expenditure is

$$\begin{aligned} E &= e_1 + e_2 + e_{31} + p \cdot e_{32} \\ &= 2\frac{c^2}{t_1} + \frac{c_1^2}{t_1} + p\frac{(c - c_1)^2}{t_2 - t_1}. \end{aligned}$$

The variables that control E are t_1 and c_1; c, t_2, and p are constants. Thus, the straightforward approach to finding the t_1 and c_1 that minimize E is to simultaneously solve the equations $dE/dt_1 = 0$ and $dE/dc_1 = 0$. These equations are, after some rearrangement,

$$p\frac{(c-c_1)^2}{(t_2-t_1)^2} = 2\frac{c^2}{t_1^2} + \frac{c_1^2}{t_1^2} \qquad p\frac{c-c_1}{t_2-t_1} = \frac{c_1}{t_1}.$$

The simultaneous solutions are

$$t_1 = \frac{\sqrt{2pq}-2q}{pq-2q^2} \times t_2 \qquad c_1 = \sqrt{\frac{2p}{q}} \times c,$$

where $q = 1 - p$. Note that $t_1 = t_2$ and $c_1 = c$ when $p = 1/3$. The expected energy utilization is

$$E = \frac{\left(2\sqrt{q}+\sqrt{2p}\right)(p-2q)}{\sqrt{2p}-2\sqrt{q}} \times \frac{c^2}{t_2}$$

$$= \frac{\left(2\sqrt{q}+\sqrt{2p}\right)^2}{2} \times \frac{c^2}{t_2}$$

4. Analysis

Figure 2 shows t_1 and c_1 as a function of p when nominally $c = 1$ and $t_2 = 1$. The figure also shows E/3 as a function of p. Its

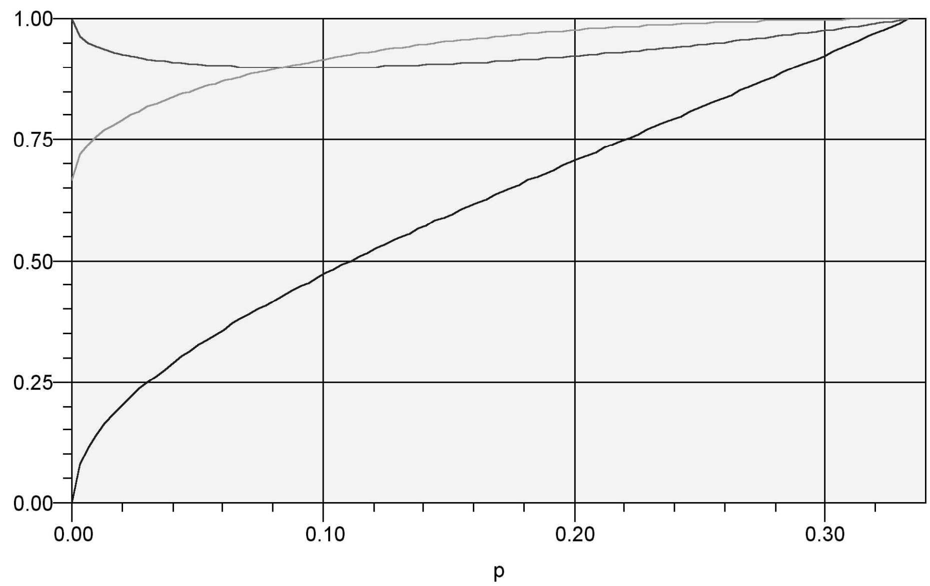

Figure 2. Plots of t_1 and c_1 versus p.

value monotonically increases from 2/3 to 1 as p goes from 0 to 1/3. Therefore, at most a 33% savings in energy is available when error-free hardware replaces the worst possible. Probably the most interesting thing to note is that the value of t_1 is always a sizable fraction of t_2. When $p = 0$, $c_1 = 0$, i.e., process 3 never executes. As p nears 1/3 the parameters make the execution of process 3 more and more like the other processes since there is a fair probability that its results will be needed.