Calibrated Critics

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Abstract

Critics provide ratings of things and these ratings tell us what the critic prefers when given choices. If we trust and value the opinions of these critics, their preferences will influence ours. This note explores some constraints between two critics and asks whether these constraints make their ratings identical. We require a calibration set of rated objects where, for every possible rating, there is an object in that set which both critics give that rating. Another constraint considered is that the critics be in general agreement which means that if one critic strictly prefers one object to another, the other critic will not have a strict preference the other way around. While calibration and general agreement do not enforce identity between two critics, a restriction on the set of ratings is consider that does.

1 Basics and Notation

Let c_1 and c_2 be two *critics* who rate the set of objects, X, with possible ratings in R. Thus, $c_i(X) = R$. We define a set $\Theta = \Theta(c_1, c_2)$ as a *calibration set* for c_1 and c_2 if $\Theta \subseteq X$ and for every $r \in R$ there is a $\theta_r \in \Theta$ such that $c_1(\theta_r) = c_2(\theta_r) = r$. If a calibration set exists for c_1 and c_2, we say that *these critics are calibrated*. The existence of such a Θ entails that c_1 and c_2 be onto R.

The relation "$<$" on R is a *strict linear order* and the critics' preference relation.[1] This means that "$<$" 1) satisfies trichotomy—for all $r_1, r_2 \in R$,

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[1]A strict linear order is similar to a linear order where the order relation resembles numeric "$<$" rather than "$\leq$".

exactly one of $r_1 < r_2$, $r_2 < r_1$, or $r_1 = r_2$ is true; and 2) is transitive—$r_1 < r_2$ and $r_2 < r_3$ implies that $r_1 < r_3$. Define "$>$", "$\geq$", "$\leq$",and "$\neq$" from "$<$" in the usual way.

The proper way to say c_1 ranks $x \in X$ higher than c_2 is $c_2(x) < c_1(x)$ or, equivalently, $c_1(x) > c_2(x)$ and the proper way to say that c_i prefers x_1 to x_2 is $c_i(x_2) < c_i(x_1)$.

We will say that c_1 and c_2 are in *general agreement* if $c_1(x_1) < c_1(x_2)$ implies $c_2(x_1) \leq c_2(x_2)$, where general agreement is a symmetric relation.

It is interesting to note that, in real life, preferences might not be transitive as is usually assumed.[2] There is nothing irrational about picking apple pie over blueberry, picking blueberry over cherry, and picking cherry over apple when given binary choices. However, we make the standard (though debateable) assumption of transitivity here so we can do a little mathematics without the intrusion of reality.

2 Is Disagreement Possible?

The question investigate in this section is: If c_1 and c_2 are in general agreement and calibrate (by a set Θ), is it possible that there is an $x \in X$ such that $c_1(x) \neq c_2(x)$? The simple answer is yes, disagreement is possible, despite the existence of calibration instances for the ratings and the fact that there can be no example pairs where each critic strictly prefers a different one of the pair.

Before I display a justifiable disagreement, I introduce a simple graphical way to display (parts of) the two critic functions c_1 and c_2. Figure 1 shows two horizontal lines labelled c_1 and c_2 that represent our two critics. Points

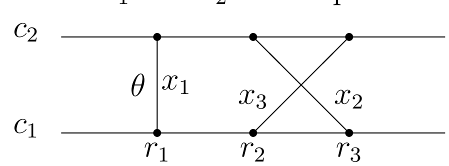

Figure 1: Graphic conventions.

on these lines are elements of R and, by convention, if label r_1 is to the left of r_2, then $r_1 < r_2$. A line between points on the two horizontal lines represents an $x \in X$; The line's intersection with the lower line is the value $c_1(x)$ and its intersection with the upper line is the value $c_2(x)$.

[2]Barnett, J.A. "Preference relations." http://notatt.com/preferences.pdf

The vertical line at r_1 shows two objects in X, namely θ and x_1, where c_1 and c_2 give the same rating, namely r_1, so are in agreement. This example illustrates the fact that multiple lines (objects) may connect the same two points on the two lines any time it is legal for one line to do so. It also illustrates the fact that there is a $\theta \in \Theta$ vertical line for *every* point on the horizontal lines whether it is shown as for r_1 or not shown as for r_2 and r_3. Recall that these figures may show only a part of the critic functions.

The two lines at r_2 and r_3 show a case where the critics are not in basic agreement: $c_1(x_3) = c_2(x_2) = r_2$ and $c_1(x_2) = c_2(x_3) = r_3$. Since $r_2 < r_3$ by convention, we have $c_1(x_3) < c_1(x_2)$ but $c_2(x_2) < c_2(x_3)$. And this is a clear case of total disagreement.

Figure 2 shows a case where the critics are in general but not exact agreement. They do agree on the ratings of θ_1 and θ_3 which are members

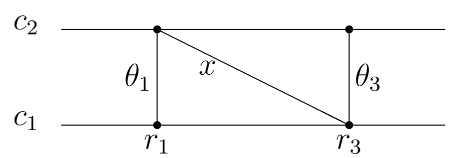

Figure 2: General but not exact agreement.

of the calibration set, Θ. However, the critics do not agree on ratings of x, where c_1 gives it a higher rating than does c_2. Though the agreement is not exact, nothing about the fragments shown of the critics, c_1 and c_2, violates any restrictions made by assuming they are calibrated or in general agreement. So we now can conclude that our assumptions cannot and do not guarantee exact agreement.

But that's not the end of the story.

3 How Dense Is *R*?

Let us modify Figure 2 by adding another rating, r_2, where $r_1 < r_2$ and $r_2 < r_3$. This modification and one to show θ_2 that calibrates the critics for an r_2 rating, are displayed in Figure 3. The new diagram shows that the critics are no longer in basic agreement because $c_1(\theta_2) < c_1(x)$ while $c_2(x) < c_2(\theta_2)$. The ingredients that produce this result are the facts that there exists a rating value between r_1 and r_3 and the existence of a calibrating example for each $r \in R$.

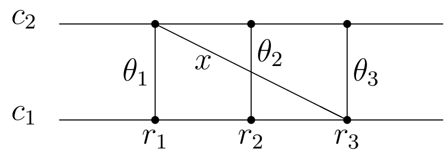

Figure 3: Another rating is added to Figure 2.

Assume that there is a strictly totally ordered set, S with the binary order relation, $\prec$. Assume further that for every $x, z \in S$, where $x \prec z$, there exists a $y \in S$ such that $x \prec y$ and $y \prec z$. In this case, "$\prec$" is said to be *dense in S*.

We therefore have a theorem: *If two critics are calibrated, in general agreement, and the order relation on ratings is dense in the set of ratings, the critics exactly agree on all objects.*

We should also note that two calibrated critics in general agreement can only disagree in ratings were there are two ratings that are not separated in value by any other rating(s).

4 Final Comments

Some sources contributing to this article were Wikipedia articles on Tarski's axiomatizations of the reals[3] and one on total orders.[4] Another article on totally ordered set is available on MathWorld.[5]

This note was inspired, in part, by a delightful article, "Calibrated Crickets" that appeared in the *Worm Runner's Digest* circa 1975.[6] It suggested putting crickets on a metal strip that was kept cold on one end and hot on the other. One then noted the temperature where the cricket preferred to stand and that was the calibration of that cricket. Crickets were to be used as a control mechanism when packages were shipped that had to be maintained at precise temperatures. A cricket calibrated to the desired temperature was used. Somehow the fact that crickets' rate of chirping indicated current temperature factored into the construction of the controller.

[3] `https://en.wikipedia.org/wiki/Tarski%27s_axiomatization_of_the_reals`

[4] `https://en.wikipedia.org/wiki/Total_order`

[5] Weisstein, Eric W. "Totally Ordered Set." From MathWorld–A Wolfram Web Resource. `https://mathworld.wolfram.com/TotallyOrderedSet.html`

[6] I could not find a precise reference to this article using Google Scholar.